

# Analysis and Evaluation of Pumping Test Data

**Second Edition (Completely Revised)**



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G.P. Kruseman

Senior hydrogeologist, TNO Institute of Applied Geoscience, Delft

N.A. de Ridder

Senior hydrogeologist, International Institute for Land Reclamation  
and Improvement, Wageningen

and

Professor in Hydrogeology, Free University, Amsterdam

With assistance from

J.M. Verweij

Freelance hydrogeologist

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# Preface

This is the second edition of *Analysis and Evaluation of Pumping Test Data*. Readers familiar with the first edition and its subsequent impressions will note a number of changes in the new edition. These changes involve the contents of the book, but not the philosophy behind it, which is to be a practical guide to all who are organizing, conducting, and interpreting pumping tests.

What changes have we made? In the first place, we have included the step-drawdown test, the slug test, and the oscillation test. We have also added three chapters on pumping tests in fractured rocks. This we have done because of comments from some of our reviewers, who regretted that the first edition contained nothing about tests in fractured rocks. It would be remiss of us, however, not to warn our readers that, in spite of the intense research that fractured rocks have undergone in the last two decades, the problem is still the subject of much debate. What we present are some of the common methods, but are aware that they are based on ideal conditions which are rarely met in nature. All the other methods, however, are so complex that one needs a computer to apply them.

We have also updated the book in the light of developments that have taken place since the first edition appeared some twenty years ago. We present, for instance, a more modern method of analyzing pumping tests in unconfined aquifers with delayed yield. We have also re-evaluated some of our earlier field examples and have added several new ones.

Another change is that, more than before, we emphasize the intricacy of analyzing field data, showing that the drawdown behaviour of totally different aquifer systems can be very similar.

It has become a common practice nowadays to use computers in the analysis of pumping tests. For this edition of our book, we seriously considered adding computer codes, but eventually decided not to because they would have made the book too voluminous and therefore too costly. Other reasons were the possible incompatibility of computer codes and, what is even worse, many of the codes are based on 'black box' methods which do not allow the quality of the field data to be checked. Interpreting a pumping test is not a matter of feeding a set of field data into a computer, tapping a few keys, and expecting the truth to appear. The only computer codes with merit are those that take over the tedious work of plotting the field data and the type curves, and display them on the screen. These computer techniques are advancing rapidly, but we have refrained from including them. Besides, the next ILRI Publication (No. 48, *SATEM: Selected Aquifer Test Evaluation Methods* by J. Boonstra) presents the most common well-flow equations in computerized form. As well, the International Ground-Water Modelling Centre in Indianapolis, U.S.A., or its branch office in Delft, The Netherlands, can provide all currently available information on computer codes.

Our wish to revise and update our book could never have been realized without the support and help of many people. We are grateful to Mr. F. Walter, Director of TNO Institute of Applied Geoscience, who made it possible for the first author and Ms

Hanneke Verwey to work on the book. We are also grateful to Brigadier (Retired) K.G. Ahmad, General Manager (Water) of the Water and Power Development Authority, Pakistan, for granting us permission to use pumping test data not officially published by his organization.

We also express our thanks to Dr J.A.H. Hendriks, Director of ILRI, who allowed the second author time to work on the book, and generously gave us the use of ILRI's facilities, including the services of Margaret Wiersma-Roche, who edited our manuscript and corrected our often wordy English. We are indebted to Betty van Aarst and Joop van Dijk for their meticulous drawings, and to Trudy Pleijsant-Paes for her patience and perseverance in processing the words and the equations of the book. Last, but by no means least, we thank ILRI's geohydrologist, Dr J. Boonstra, for his discussion of the three chapters on fractured rocks and his valuable contribution to their final draft.

We hope that this revised and updated edition of *Analysis and Evaluation of Pumping Test Data* will serve its readers as the first edition did. Any comments anyone would care to make will be received with great interest.

G.P. Kruseman  
N.A. de Ridder

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# 1 Basic concepts and definitions

When working on problems of groundwater flow, the geologist or engineer has to find reliable values for the hydraulic characteristics of the geological formations through which the groundwater is moving. Pumping tests have proved to be one of the most effective ways of obtaining such values.

Analyzing and evaluating pumping test data, however, is as much an art as a science. It is a science because it is based on theoretical models that the geologist or engineer must understand and on thorough investigations that he must conduct into the geological formations in the area of the test. It is an art because different types of aquifers can exhibit similar drawdown behaviours, which demand interpretational skills on the part of the geologist or engineer. We hope that this book will serve as a guide in both the science and the art.

The equations we present in this book are from well hydraulics. We have omitted any lengthy derivations of the equations because these can be found in the original publications listed in our References. With some exceptions, we present the equations in their final form, emphasizing the assumptions and conditions that underlie them, and outlining the procedures that are to be followed for their successful application.

'Hard rocks', both as potential sources of water and depositories for chemical or radioactive wastes, are receiving increasing attention in hydrogeology. We shall therefore be discussing some recent developments in the interpretation of pumping test data from such rocks.

This chapter summarizes the basic concepts and definitions of terms relevant to our subject. The next chapter describes how to conduct a pumping test. The remaining chapters all deal with the analysis and evaluation of pumping test data from a variety of aquifer types or aquifer systems, and from tests conducted under particular technical conditions.

## 1.1 Aquifer, aquitard, and aquiclude

An aquifer is defined as a saturated permeable geological unit that is permeable enough to yield economic quantities of water to wells. The most common aquifers are unconsolidated sand and gravels, but permeable sedimentary rocks such as sandstone and limestone, and heavily fractured or weathered volcanic and crystalline rocks can also be classified as aquifers.

An aquitard is a geological unit that is permeable enough to transmit water in significant quantities when viewed over large areas and long periods, but its permeability is not sufficient to justify production wells being placed in it. Clays, loams and shales are typical aquitards.

An aquiclude is an impermeable geological unit that does not transmit water at all. Dense unfractured igneous or metamorphic rocks are typical aquicludes. In nature, truly impermeable geological units seldom occur; all of them leak to some extent, and must therefore be classified as aquitards. In practice, however, geological units

can be classified as aquicludes when their permeability is several orders of magnitude lower than that of an overlying or underlying aquifer.

The reader will note that the above definitions are relative ones; they are purposely imprecise with respect to permeability.

## 1.2 Aquifer types

There are three main types of aquifer: confined, unconfined, and leaky (Figure 1.1).

### 1.2.1 Confined aquifer

A confined aquifer (Figure 1.1A) is bounded above and below by an aquiclude. In a confined aquifer, the pressure of the water is usually higher than that of the atmosphere, so that if a well taps the aquifer, the water in it stands above the top of the aquifer, or even above the ground surface. We then speak of a free-flowing or artesian well.

### 1.2.2 Unconfined aquifer

An unconfined aquifer (Figure 1.1B), also known as a watertable aquifer, is bounded below by an aquiclude, but is not restricted by any confining layer above it. Its upper boundary is the watertable, which is free to rise and fall. Water in a well penetrating an unconfined aquifer is at atmospheric pressure and does not rise above the watertable.

### 1.2.3 Leaky aquifer

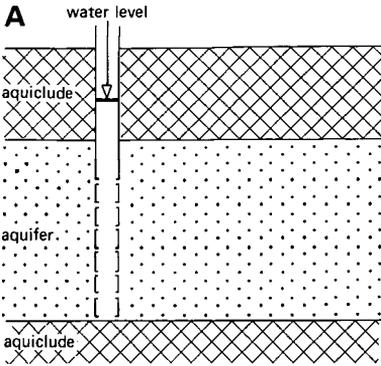
A leaky aquifer (Figure 1.1C and D), also known as a semi-confined aquifer, is an aquifer whose upper and lower boundaries are aquitards, or one boundary is an aquitard and the other is an aquiclude. Water is free to move through the aquitards, either upward or downward. If a leaky aquifer is in hydrological equilibrium, the water level in a well tapping it may coincide with the watertable. The water level may also stand above or below the watertable, depending on the recharge and discharge conditions.

In deep sedimentary basins, an interbedded system of permeable and less permeable layers that form a multi-layered aquifer system (Figure 1.1E), is very common. But such an aquifer system is more a succession of leaky aquifers, separated by aquitards, rather than a main aquifer type.

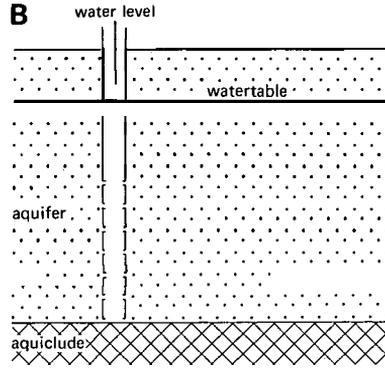
## 1.3 Anisotropy and heterogeneity

Most well hydraulics equations are based on the assumption that aquifers and aquitards are homogeneous and isotropic. This means that the hydraulic conductivity is the same throughout the geological formation and is the same in all directions (Figure

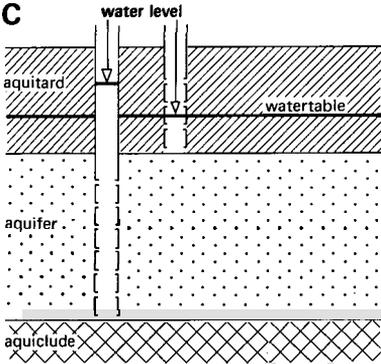
**CONFINED AQUIFER**



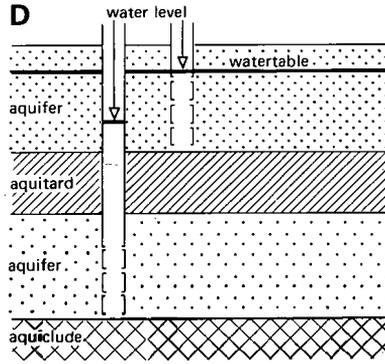
**UNCONFINED AQUIFER**



**LEAKY AQUIFER**



**LEAKY AQUIFER**



**MULTI-LAYERED LEAKY AQUIFER SYSTEM**

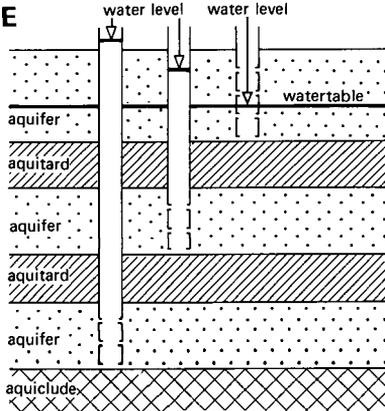


Figure 1.1 Different types of aquifers  
A. Confined aquifer  
B. Unconfined aquifer  
C. and D. Leaky aquifers  
E. Multi-layered leaky aquifer

1.2A). The individual particles of a geological formation, however, are seldom spherical so that, when deposited under water, they tend to settle on their flat sides. Such a formation can still be homogeneous, but its hydraulic conductivity in horizontal direction,  $K_h$ , will be significantly greater than its hydraulic conductivity in vertical direction,  $K_v$  (Figure 1.2B). This phenomenon is called anisotropy.

The lithology of most geological formations tends to vary significantly, both horizontally and vertically. Consequently, geological formations are seldom homogeneous. Figure 1.2C is an example of layered heterogeneity. Heterogeneity occurs not only in the way shown in the figure: individual layers may pinch out; their grain size may vary in horizontal direction; they may contain lenses of other grain sizes; or they may be discontinuous by faulting or scour-and-fill structures. In horizontally-stratified alluvial formations, the  $K_h/K_v$  ratios range from 2 to 10, but values as high as 100 can occur, especially where clay layers are present.

Anisotropy is a common property of fractured rocks (Figure 1.2D). The hydraulic conductivity in the direction of the main fractures is usually significantly greater than that normal to those fractures.

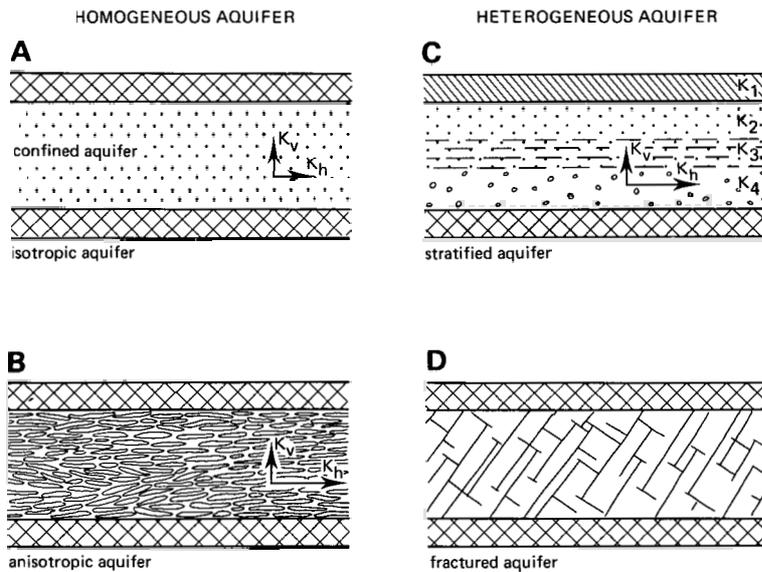


Figure 1.2 Homogeneous and heterogeneous aquifers, isotropic and anisotropic

- A. Homogeneous aquifer, isotropic
- B. Homogeneous aquifer, anisotropic
- C. Heterogeneous aquifer, stratified
- D. Heterogeneous aquifer, fractured

If the principal directions of anisotropy are known, one can transform an anisotropic system into an isotropic system by changing the coordinates. In the new coordinate system, the basic well-flow equation is again isotropic and the common equations can be used.

## 1.4 Bounded aquifers

Another common assumption in well hydraulics is that the pumped aquifer is horizontal and of infinite extent. But, viewed on a regional scale, some aquifers slope, and none of them extend to infinity because complex geological processes cause interfingering of layers and pinchouts of both aquifers and aquitards. At some places, aquifers and aquitards are cut by deeply incised channels, estuaries, or the ocean. In other words, aquifers and aquitards are laterally bounded in one way or another. Figure 1.3 shows some examples. The interpretation of pumping tests conducted in the vicinity of such boundaries requires special techniques, which we shall be discussing.

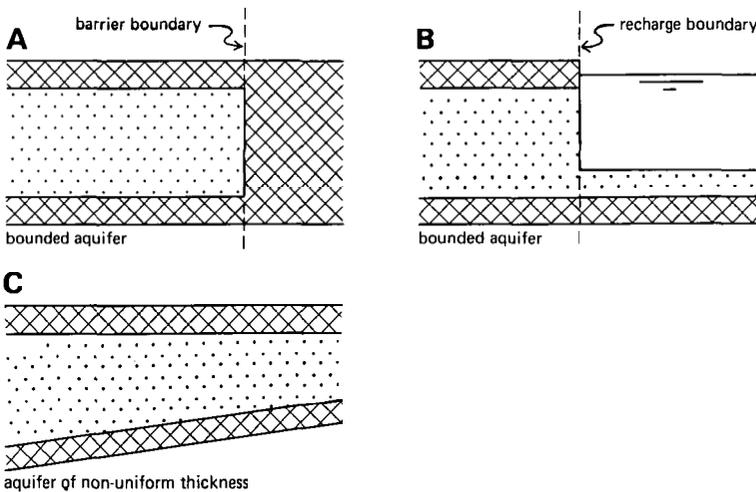


Figure 1.3 Bounded aquifers  
A, B, and C

## 1.5 Steady and unsteady flow

There are two types of well-hydraulics equations: those that describe steady-state flow towards a pumped well and those that describe the unsteady-state flow.

Steady-state flow is independent of time. This means that the water level in the pumped well and in surrounding piezometers does not change with time. Steady-state flow occurs, for instance, when the pumped aquifer is recharged by an outside source, which may be rainfall, leakage through aquitards from overlying and/or underlying unpumped aquifers, or from a body of open water that is in direct hydraulic contact

with the pumped aquifer. In practice, it is said that steady-state flow is attained if the changes in the water level in the well and piezometers have become so small with time that they can be neglected. As pumping continues, the water level may drop further, but the hydraulic gradient induced by the pumping will not change. In other words, the flow towards the well has attained a pseudo-steady-state.

In well hydraulics of fractured aquifers, the term pseudo-steady-state is used for the interporosity flow from the matrix blocks to the fractures. This flow occurs in response to the difference between the average hydraulic head in the matrix blocks and the average hydraulic head in the fractures. Spatial variation in hydraulic head gradients in the matrix blocks is ignored and the flow through the fractures to the well is radial and unsteady.

Unsteady-state flow occurs from the moment pumping starts until steady-state flow is reached. Consequently, if an infinite, horizontal, completely confined aquifer of constant thickness is pumped at a constant rate, there will always be unsteady-state flow. In practice, the flow is considered to be unsteady as long as the changes in water level in the well and piezometers are measurable or, in other words, as long as the hydraulic gradient is changing in a measurable way.

## 1.6 Darcy's law

Darcy's law states that the rate of flow through a porous medium is proportional to the loss of head, and inversely proportional to the length of the flow path, or

$$v = K \frac{\Delta h}{\Delta l} \quad (1.1)$$

or, in differential form

$$v = K \frac{dh}{dl} \quad (1.2)$$

where  $v = Q/A$ , which is the specific discharge, also known as the Darcy velocity or Darcy flux (Length/Time),  $Q =$  volume rate of flow (Length<sup>3</sup>/Time),  $A =$  cross-sectional area normal to flow direction (Length<sup>2</sup>),  $\Delta h = h_2 - h_1$ , which is the head loss, whereby  $h_1$  and  $h_2$  are the hydraulic heads measured at Points 1 and 2 (Length),  $\Delta l =$  the distance between Points 1 and 2 (Length),  $dh/dl = i$ , which is the hydraulic gradient (dimensionless), and  $K =$  constant of proportionality known as the hydraulic conductivity (Length/Time).

Alternatively, Darcy's law can be written as

$$Q = K \frac{dh}{dl} A \quad (1.3)$$

Note that the specific discharge  $v$  has the dimensions of a velocity, i.e. Length/Time. The concept specific discharge assumes that the water is moving through the entire porous medium, solid particles as well as pores, and is thus a macroscopic concept. The great advantage of this concept is that the specific discharge can be easily measured. It must, however, be clearly differentiated from the microscopic velocities, which are real velocities. Hence, if we are interested in real flow velocities, as in prob-

lems of groundwater pollution and solute transport, we must consider the actual paths of individual water particles as they find their way through the pores of the medium. In other words, we must consider the porosity of the transmitting medium and can write

$$v_a = \frac{v}{n} \text{ or } v_a = \frac{Q}{nA} \quad (1.4)$$

where  $v_a$  = real velocity of the flow, and  $n$  = porosity of the water-transmitting medium.

In using Darcy's law, one must know the range of its validity. After all, Darcy (1856) conducted his experiments on sand samples in the laboratory. So, Darcy's law is valid for laminar flow, but not for turbulent flow, as may happen in cavernous limestone or fractured basalt. In case of doubt, one can use the Reynolds number as a criterion to distinguish between laminar and turbulent flow. The Reynolds number is expressed as

$$N_R = \rho \frac{vd}{\mu} \quad (1.5)$$

where  $\rho$  is the fluid density,  $v$  is the specific discharge,  $\mu$  is the viscosity of the fluid, and  $d$  is a representative length dimension of the porous medium, usually taken as a mean grain diameter or a mean pore diameter.

Experiments have shown that Darcy's law is valid for  $N_R < 1$  and that no serious errors are created up to  $N_R = 10$ . This value thus represents an upper limit to the validity of Darcy's law. It should not be considered a unique limit, however, because turbulence occurs gradually. At full turbulence ( $N_R < 100$ ), the head loss varies approximately with the second power of the velocity rather than linearly. Fortunately, most groundwater flow occurs with  $N_R < 1$  so that Darcy's law applies. Only in exceptional situations, as in a rock with wide openings, or where steep hydraulic gradients exist, as in the near vicinity of a pumped well, will the criterion of laminar flow not be satisfied and Darcy's law will be invalid.

Darcy's law is also invalid at low hydraulic gradients, as may occur in compact clays, because, for low values of  $i$ , the relation between  $v$  and  $i$  is not linear. It is impossible to give a unique lower limit to the hydraulic gradients at which Darcy's law is still valid, because the values of  $i$  vary with the type and structure of the clay, while the mineral content of the water also plays a role (De Marsily 1986).

## 1.7 Physical properties

In the equations describing the flow to a pumped well, various physical properties and parameters of aquifers and aquitards appear. These will be discussed below.

### 1.7.1 Porosity ( $n$ )

The porosity of a rock is its property of containing pores or voids. If we divide the total unit volume  $V_T$  of an unconsolidated material into the volume of its solid portion

$V_s$  and the volume of its voids  $V_v$ , we can define the porosity as  $n = V_v/V_T$ . Porosity is usually expressed as a decimal fraction or as a percentage.

With consolidated and hard rocks, a distinction is usually made between primary porosity, which is present when the rock is formed, and secondary porosity, which develops later as a result of solution or fracturing. As Figure 1.4 shows, fractures

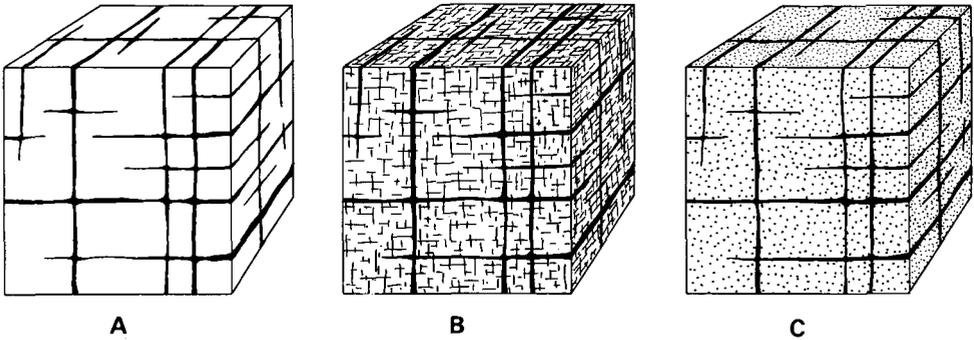


Figure 1.4 Porosity systems  
 A. Single porosity  
 B. Microfractures  
 C. Double porosity

can be oriented in three main directions, which cut the rock into blocks. In theory, the primary porosity of a dense solid rock may be zero and the rock matrix will be impermeable. Such a rock can be regarded as a single-porosity system (Figure 1.4A). In some rocks, notably crystalline rocks, the main fractures are accompanied by a dense system of microfractures, which considerably increase the porosity of the rock matrix (Figure 1.4B). In contrast, the primary porosity of granular geological formations (e.g. sandstone) can be quite significant (Figure 1.4C). When such a formation is fractured, it can be regarded as a double-porosity system because the two types of porosities coexist: the primary or matrix porosity and the secondary or fracture porosity.

Table 1.1 gives some porosity values for unconsolidated materials and rocks.

Table 1.1 Range of porosity values (n) in percentages

Rocks		Unconsolidated materials	
Sandstone	5–30	Gravel	25–40
Limestone	0–20	Sand	25–50
Karstic limestone	5–50	Silt	35–50
Shale	0–10	Clay	40–70
Basalt, fractured	5–50		
Crystalline rock	0–5		
Crystalline rock, fractured	0–10		

### 1.7.2 Hydraulic conductivity (K)

The hydraulic conductivity is the constant of proportionality in Darcy's law (Equation 1.3). It is defined as the volume of water that will move through a porous medium in unit time under a unit hydraulic gradient through a unit area measured at right angles to the direction of flow. Hydraulic conductivity can have any units of Length/Time, for example m/d.

The hydraulic conductivity of fractured rocks depends largely on the density of the fractures and the width of their apertures. Fractures can increase the hydraulic conductivity of solid rocks by several orders of magnitude.

The significant effect that fractures can have on the hydraulic conductivity of hard rocks has been treated by various authors. Maini and Hocking (1977), for example, as quoted by De Marsily (1986), give the equivalence between the hydraulic conductivity of a fractured rock and that of a porous (granular) aquifer. From their diagram, it follows that the flow through, say, a 100 m thick cross-section of a porous medium with a hydraulic conductivity of  $10^{-12}$  m/d could, in a fractured medium with an impermeable rock matrix, also come from one single fracture only 0.2 mm wide.

For orders of magnitude of K for different materials, see Table 1.2.

Table 1.2 Order of magnitude of K for different kinds of rock (from Bouwer 1978)

Geological classification	K (m/d)	
Unconsolidated materials:		
Clay	$10^{-8}$	$- 10^{-2}$
Fine sand	1	- 5
Medium sand	5	$- 2 \times 10^1$
Coarse sand	$2 \times 10^1$	$- 10^2$
Gravel	$10^2$	$- 10^3$
Sand and gravel mixes	5	$- 10^2$
Clay, sand, gravel mixes (e.g. till)	$10^{-3}$	$- 10^{-1}$
Rocks:		
Sandstone	$10^{-3}$	-1
Carbonate rock with secondary porosity	$10^{-2}$	-1
Shale	$10^{-7}$	
Dense solid rock	$< 10^{-5}$	
Fractured or weathered rock (Core samples)	Almost 0 - $3 \times 10^2$	
Volcanic rock	Almost 0 - $10^3$	

### 1.7.3 Interporosity flow coefficient ( $\lambda$ )

When a confined fractured aquifer of the double-porosity type is pumped, the interporosity flow coefficient controls the flow in the aquifer. It indicates how easily water can flow from the aquifer matrix blocks into the fractures, and is defined as

$$\lambda = \alpha r^2 \frac{K_m}{K_f} \quad (1.6)$$

where  $\alpha$  is a shape factor that reflects the geometry of the matrix blocks,  $r$  is the distance to the well,  $K$  is hydraulic conductivity,  $f$  is the fracture, and  $m$  is matrix block. The dimension of  $\lambda$  is reciprocal area.

#### 1.7.4 Compressibility ( $\alpha$ and $\beta$ )

Compressibility is an important material and fluid property in the analysis of unsteady flow to wells. It describes the change in volume or the strain induced in an aquifer (or aquitard) under a given stress, or

$$\alpha = \frac{-dV_T/V_T}{d\sigma_e} \quad (1.7)$$

where  $V_T$  is the total volume of a given mass of material and  $d\sigma_e$  is the change in effective stress. Compressibility is expressed in  $m^2/N$  or  $Pa^{-1}$ . Its value for clay ranges from  $10^{-6}$  to  $10^{-8}$ , for sand from  $10^{-7}$  to  $10^{-9}$ , for gravel and fractured rock from  $10^{-8}$  to  $10^{-10} m^2/N$ .

Similarly, the compressibility of water is defined as

$$\beta = \frac{-dV_w/V_w}{dp} \quad (1.8)$$

A change in the water pressure  $dp$  induces a change in the volume  $V_w$  of a given mass of water. The compressibility of groundwater under the range of temperatures that are usually encountered can be taken constant as  $4.4 \times 10^{-10} m^2/N$  (or  $Pa^{-1}$ ).

#### 1.7.5 Transmissivity (KD or T)

Transmissivity is the product of the average hydraulic conductivity  $K$  and the saturated thickness of the aquifer  $D$ . Consequently, transmissivity is the rate of flow under a unit hydraulic gradient through a cross-section of unit width over the whole saturated thickness of the aquifer. The effective transmissivity, as used for fractured media, is defined as

$$T = \sqrt{T_{f(x)}T_{f(y)}} \quad (1.9)$$

where  $f$  refers to the fractures and  $x$  and  $y$  to the principal axes of permeability. Transmissivity has the dimensions of  $Length^3/Time \times Length$  or  $Length^2/Time$  and is, for example, expressed in  $m^2/d$  or  $m^2/s$ .

#### 1.7.6 Specific storage ( $S_s$ )

The specific storage of a saturated confined aquifer is the volume of water that a unit volume of aquifer releases from storage under a unit decline in hydraulic head. This release of water from storage under conditions of decreasing head  $h$  stems from the compaction of the aquifer due to increasing effective stress  $\sigma_e$  and the expansion

of the water due to decreasing pressure  $p$ . Hence, the earlier-defined compressibilities of material and water play a role in these two mechanisms. The specific storage is defined as

$$S_s = \rho g(\alpha + n\beta) \quad (1.10)$$

where  $\rho$  is the mass density of water ( $M/L^3$ ),  $g$  is the acceleration due to gravity ( $N/L^3$ ), and the other symbols are as defined earlier. The dimension of specific storage is  $Length^{-1}$ .

### 1.7.7 Storativity (S)

The storativity of a saturated confined aquifer of thickness  $D$  is the volume of water released from storage per unit surface area of the aquifer per unit decline in the component of hydraulic head normal to that surface. In a vertical column of unit area extending through the confined aquifer, the storativity  $S$  equals the volume of water released from the aquifer when the piezometric surface drops over a unit distance. Storativity is defined as

$$S = \rho g D(\alpha + n\beta) = S_s D \quad (1.11)$$

As storativity involves a volume of water per volume of aquifer, it is a dimensionless quantity. Its values in confined aquifers range from  $5 \times 10^{-5}$  to  $5 \times 10^{-3}$ .

### 1.7.8 Storativity ratio ( $\omega$ )

The storativity ratio is a parameter that controls the flow from the aquifer matrix blocks into the fractures of a confined fractured aquifer of the double-porosity type. (See also Sections 1.7.1 and 1.7.3.) It is defined as

$$\omega = \frac{S_f}{S_f + S_m} \quad (1.12)$$

where  $S$  is the storativity and  $f$  is fracture and  $m$  is matrix block. Being a ratio,  $\omega$  is dimensionless.

### 1.7.9 Specific yield ( $S_y$ )

The specific yield is the volume of water that an unconfined aquifer releases from storage per unit surface area of aquifer per unit decline of the watertable. The values of the specific yield range from 0.01 to 0.30 and are much higher than the storativities of confined aquifers. In unconfined aquifers, the effects of the elasticity of the aquifer matrix and of the water are generally negligible. Specific yield is sometimes called effective porosity, unconfined storativity, or drainable pore space. Small interstices do not contribute to the effective porosity because the retention forces in them are greater than the weight of water. Hence, no groundwater will be released from small interstices by gravity drainage.

It is obvious that water can only move through pores that are interconnected. Hard rocks may contain numerous unconnected pores in which the water is stagnant. The most common example is that of secondary dolomite. Dolomitization increases the porosity because the diagenetic transformation of calcite into dolomite is accompanied by a 13% reduction in volume of the rock (Matthess 1982). The porosity of secondary dolomite is high, 20 to 30%, but the effective porosity is low because the pores are seldom interconnected. Water in 'dead-end' pores is also almost stagnant, so such pores are excluded from the effective porosity. They do play a role, of course, when one is studying the mechanisms of compressibility and solute transport in porous media.

In fractured rocks, water only moves through the fractures, even if the unfractured matrix blocks are porous. This means that the effective porosity of the rock mass is linked to the volume of these fractures. A fractured granite, for example, has a matrix porosity of 1 to 2 %, but its effective porosity is less than 1 % because the matrix itself has a very low permeability (De Marsily 1986).

Table 1.3 gives some representative values of specific yields for different materials.

Table 1.3 Representative values of specific yield (Johnson 1967)

Material	$S_y$	Material	$S_y$
Coarse gravel	23	Limestone	14
Medium gravel	24	Dune sand	38
Fine gravel	25	Loess	18
Coarse sand	27	Peat	44
Medium sand	28	Schist	26
Fine sand	23	Siltstone	12
Silt	8	Silty till	6
Clay	3	Sandy till	16
Fine-grained sandstone	21	Gravelly till	16
Medium-grained sandstone	27	Tuff	21

#### 1.7.10 Diffusivity (KD/S)

The hydraulic diffusivity is the ratio of the transmissivity and the storativity of a saturated aquifer. It governs the propagation of changes in hydraulic head in the aquifer. Diffusivity has the dimension of Length<sup>2</sup>/Time.

#### 1.7.11 Hydraulic resistance (c)

The hydraulic resistance characterizes the resistance of an aquitard to vertical flow, either upward or downward. It is the reciprocal of the leakage or leakage coefficient  $K'/D'$  in Darcy's law when this law is used to characterize the amount of leakage through the aquitard;  $K'$  = the hydraulic conductivity of the aquitard for vertical flow, and  $D'$  = the thickness of the aquitard. The hydraulic resistance is thus defined as

$$c = \frac{D'}{K'} \quad (1.13)$$

and has the dimension of Time. It is often expressed in days. Values of  $c$  vary widely, from some hundreds of days to several ten thousand days; for aquicludes,  $c$  is infinite.

#### 1.7.12 Leakage factor (L)

The leakage factor, or characteristic length, is a measure for the spatial distribution of the leakage through an aquitard into a leaky aquifer and vice versa. It is defined as

$$L = \sqrt{KDc} \quad (1.14)$$

Large values of  $L$  indicate a low leakage rate through the aquitard, whereas small values of  $L$  mean a high leakage rate. The leakage factor has the dimension of Length, expressed, for example, in metres.

