4 Theory of Soil-Water-Balance Model SWATRE

4.1 SWATRE General

For the flow below the basin area (Figure 4.1), we can restrict ourselves to vertical flow, as described by Darcy's Law:

$$q = -K(h)[(\partial h/\partial z) + 1] \tag{59}$$

where q is the flux density positive upwards ($L^3L^{-2}T^{-1}$), $K(h)$ is the hydraulic conductivity (LT^{-1}), h is the soil-water pressure head (L), and z is the vertical coordinate, origin at soil surface of basin bottom, *positive upwards* (L).

The change in stored soil water, W, with depth is:

$$\frac{\partial}{\partial z}\left[\frac{\partial W}{\partial t}\right] = \frac{\partial \theta}{\partial t} = \frac{\partial \theta}{\partial h}\frac{\partial h}{\partial t} = C\frac{\partial h}{\partial t} \tag{60}$$

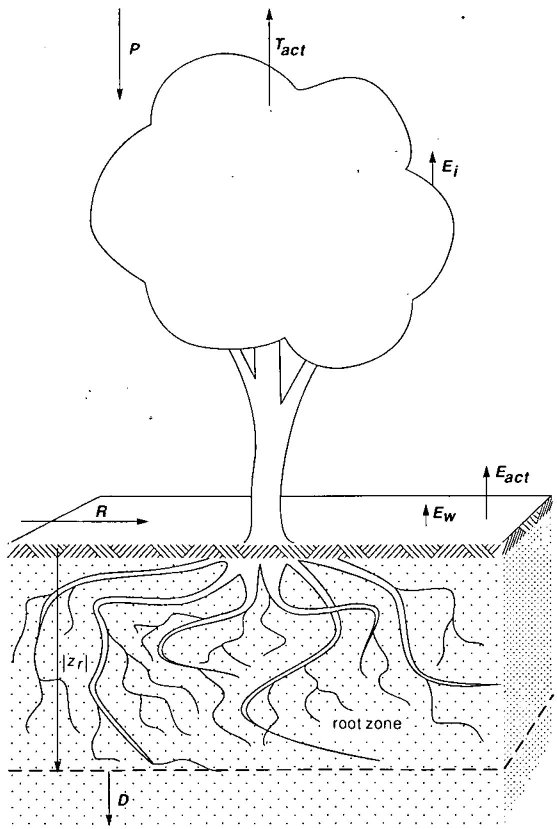

Figure 4.1 Basin area of micro-catchment with a tree and water-balance components as discussed for Equation 1.

where W is the soil water storage in the rootzone (L), θ is the volumetric soil water content (–), $C = d\theta/dh$ is the differential soil-water capacity (L^{-1}), and t is time (T).

The conservation of mass requires that:

$$C[\partial h/\partial t] = -[\partial q/\partial z] - S \tag{61}$$

where S is the volume of water taken up by the roots per unit bulk volume of soil per unit time ($L^3L^{-3}T^{-1}$).

Combining Equations 59 and 61 yields:

$$\frac{\partial h}{\partial t} = \frac{1}{C(h)}\frac{\partial}{\partial z}\left[K(h)\left(\frac{\partial h}{\partial z} + 1\right)\right] - \frac{S(h)}{C(h)} \tag{62}$$

S is expressed as:

$$S(h) = \alpha(h)S_{\max} \tag{63}$$

where $\alpha(h)$ is a prescribed function of soil-water pressure head (–), and S_{max} is the maximum possible root extraction rate (T^{-1}).

If α equals 1, the transpiration rate is completely controlled by atmospheric conditions; if $0 < \alpha < 1$, the soil-water status becomes important (Figure 4.2). For $h_1 \leq h < h_3$, $S(h)$ is expressed according to Feddes et al. (1978). From h_3 to h_4, $S(h)$ reduces exponentially according to Wesseling et al. (1989)

Equation 62 can be solved numerically. Feddes et al. (1978) and Belmans et al. (1983) developed a *transient 1-D finite difference model SWATRE,* which can be used for a wide range of boundary conditions, when solving problems of saturated or unsaturated flow. To obtain a unique solution of Equation 62, initial and boundary conditions must be specified.

The following discussion of the model theory is split into two separate sections. The application to the extremely arid and arid zones in the Negev Desert is based on the model calibration at an experimental field in Sede Boqer with Pistachio trees, where many measured data were available (see Chapter 5). The application to the semi-arid zones in Niger and Nigeria is based on the model calibration at an experimental Neem windbreak in Sadoré, Niger, for which fewer data were available (see Chapter 7).

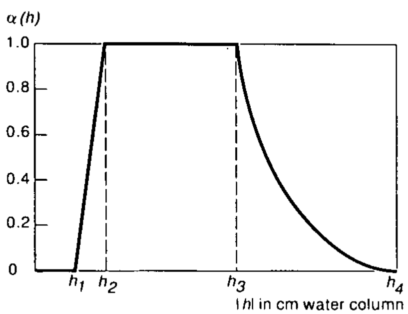

Figure 4.2 Shape of the sink term used to describe the soil-water extraction pattern. Between pressure head h_3 and h_4, the relation is exponential and the value of the exponent is -2 (Wesseling 1989).

For the application of SWATRE to Negev, Niger, and Nigeria, model options were selected that were most applicable to tree characteristics and data availability. Tree characteristics that differ for Pistachio and Neem are root distribution and root water uptake, leaves falling in winter (Pistachio trees), or year-round canopy (Neem is evergreen). Because of data availability, the determination of parameter values and initial and boundary conditions were different. For this reason, the theory will be discussed in *two separate sections*.

4.2 SWATRE Applied to the Negev Desert

The Pistachio trees in the experimental field at Sede Boqer had a rather shallow root system (see Chapter 5). The distribution of root mass and *root water uptake* was assumed to be uniform with depth (Figure 4.3). S_{max} was defined as:

$$S_{\max} = T_{\max}/|z_r| \qquad (64)$$

where T_{max} is the maximum possible transpiration rate (LT^{-1}), and $|z_r|$ is the depth of the rootzone (L).

T_{max} in Equation 64 was calculated from:

$$T_{\max} = K_c E_{\text{pan}} \qquad (65)$$

where K_c is a crop coefficient (–), and E_{pan} is the Class A pan evaporation rate (LT^{-1}).

The *initial condition* is the pressure head as a function of z, specified according to:

$$h(z,0) = h_0 \qquad (66)$$

where h_0 is the prescribed pressure head (L).

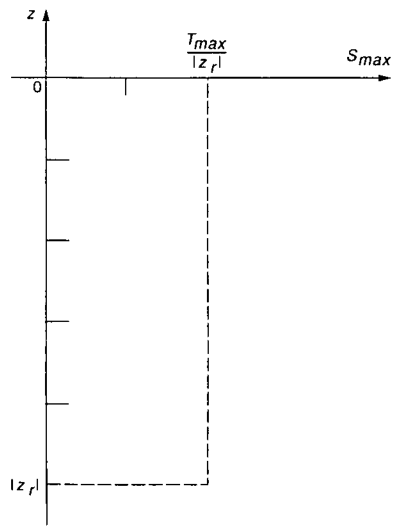

Figure 4.3 Root water uptake pattern assumed constant with depth according to Feddes et al. (1978).

The *lower boundary condition* at the bottom of the soil profile, in the absence of a groundwater table, is described as $\partial h/\partial z = 0$, which means free percolation. For this case, Equation 59 reduces to:

$$q(z=-z_b,t) = -K(h) \tag{67}$$

where $|z_b|$ is the depth of the soil profile (L).

As *upper boundary condition*, a flux density at the surface is used. The flux density through the soil surface $q_s(z=0,t)$, as determined by atmospheric conditions, is calculated as:

$$q_s(z=0,t) = E_s - p_I \tag{68}$$

where p_I is the rain storm intensity (LT^{-1}), i.e. rate of rainfall in one day (cm d^{-1}). For any day in a dry period, the *soil evaporation rate* E_s (LT^{-1}) in Equation 68 was estimated according to Black et al. (1969):

$$E_s = \lambda\sqrt{(t+1)} - \lambda\sqrt{t} \tag{69}$$

With the restriction:

$$E_s < E_{max} = K_e E_{pan} \tag{70}$$

where λ is a soil-dependent parameter ($LT^{-3/2}$), t is the time after the start of a dry period (T) (a dry period ends on the day after which $P > 0.5$ cm d^{-1}), E_{max} is the maximum possible soil evaporation rate (LT^{-1}), and K_e is a soil evaporation factor (–).

The flux density through the soil surface $q(z=0,t)$ is also governed by the transmitting properties of the top soil layer, which can be calculated according to Darcy (Equation 59). During evaporation, the pressure head at the soil surface, h_0, is assumed to be in equilibrium with the surrounding atmosphere. During infiltration, $h_0 = 0$. Actual evaporation/infiltration flux density is taken as the minimum of q_s according to Equation 68, and $q(z=0,t)$ according to Equation 59.

In Equation 68, p_I is the rain storm intensity. Evaporation of *intercepted rainwater* was neglected, since the trees have no leaves during the rainy season, which coincides with the winter. The *open water evaporation* from the basin area during infiltration was also neglected because of the brief period in which it occurs, in the order of a day.

4.3 SWATRE Applied to Niger and Nigeria

The Neem trees in the experimental windbreak at Sadoré, Niger, were assumed to have a root system with shallow horizontally spread roots and a deep tap root. This type of root distribution is often developed in arid environments by trees such as Neem or Eucalyptus. The assumption was made that most of the water is withdrawn from the upper layer, and that *soil-water withdrawal* decreases linearly with depth (Figure 4.4). S_{max} was defined according to Prasad (1988):

$$S_{max}(z) = \frac{2T_{pot}}{|z_r|}\left(1 - \frac{|z|}{|z_r|}\right) \tag{71}$$

where T_{pot} is the potential transpiration rate of the tree (LT^{-1}).

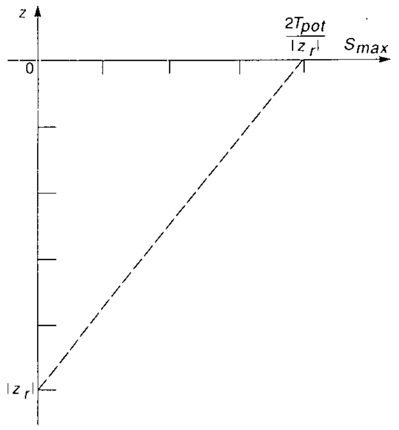

Figure 4.4 Root water uptake pattern, assumed decreasing with depth according to Prasad (1988).

Unlike Sede Boqer, where data were available to relate T_{max} in Equation 64 directly to E_{pan} in Equation 65, data were not available for Sadoré, Niger. Therefore, T_{pot} in Equation 71 was determined as follows:

$$T_{\text{pot}} = ET_{\text{pot}} - E_{\text{pot}} \tag{72}$$

where ET_{pot} is the potential evapotranspiration rate of the tree (LT^{-1}), and E_{pot} is the potential soil evaporation rate of shaded soil, i.e. bare soil not receiving any radiation under the evergreen closed windbreak canopy (LT^{-1}).

The potential evapotranspiration rate of the tree ET_{pot} was:

$$ET_{\text{pot}} = K_{\text{tree}} ET_0 \tag{73}$$

where K_{tree} is the crop factor for the tree (–), and ET_0 is the evapotranspiration rate of a reference crop (LT^{-1}) according to Doorenbos and Pruitt (1977).

The potential soil evaporation rate E_{pot} was calculated as:

$$E_{\text{pot}} = K_{\text{soil}} ET_0 \tag{74}$$

where K_{soil} is the soil evaporation factor of bare soil shaded by the windbreak canopy (–).

The evapotranspiration rate of the reference crop ET_0 was found from:

$$ET_0 = K_{\text{pan}} E_{\text{pan}} \tag{75}$$

where K_{pan} is the pan evaporation factor (–).

The *initial condition* was defined as:

$$\theta(z,0) = \theta_0 \tag{76}$$

where θ_0 is the prescribed volumetric soil water content (–).

In the absence of measured θ-values, the θ_0-value was estimated by a procedure described in Chapter 7.

The *lower boundary condition* at the bottom of the soil profile, in the absence of a groundwater table, is free percolation: $\partial h/\partial z = 0$, with Equation 59 reduced to Equation 67.

As *upper boundary condition*, a flux density at the surface was used. The flux density through the soil surface $q_s(z=0,t)$, as determined by atmospheric conditions, was calculated as:

$$q_s(z=0,t) = E_{soil} - p_{I,n} \tag{77}$$

where E_{soil} is the soil evaporation rate under the windbreak canopy (LT^{-1}), and $p_{I,n}$ is the net rainstorm intensity (LT^{-1}), i.e. the rate of rainfall minus water interception in one day (cm d^{-1}).

The soil evaporation rate under the Neem windbreak canopy, E_{soil} in Equation 77, is different from E_s under one Pistachio tree in Equation 68 because of the prevailing conditions. In the Negev desert, rainfall occurs in the winter season, when the *Pistachio trees have no leaves,* and the wet soil in the basin area *is exposed to radiation.* Soil evaporation occurs mainly in this season. During the following growing season, the soil has already dried out in preceding dry spells and, in addition, the tree canopy provides shade to the soil surface.

In Niger, the *Neem trees are evergreen* and the *windbreak canopy is closed.* Rainfall occurs in the summer season, but the *soil is never exposed to direct radiation,* and so the soil evaporation rate in the shade of the windbreak canopy is limited.

At Sede Boqer, data were available to determine the soil evaporation parameter λ in Equation 69, and to relate E_{max} to E_{pan} in Equation 70. These data were not available for Sadoré, and E_{soil} in Equation 77 was assumed to be equal to E_{pot} in Equation 74. This E_{pot} was estimated low by applying a low value of K_{soil}, lower than that given by Doorenbos and Pruitt (1977) for bare field soil exposed to radiation.

The *open water evaporation* from the basin during infiltration was assumed to be negligible for two reasons. First, the infiltration usually takes a relatively short time, and second, in the shade of the canopy, open water evaporation is low.

Evaporation of intercepted rainwater on the leaves of the evergreen Neem windbreak cannot be neglected during summer when evaporation rates are high. Equation 77 takes into account the interception loss in net rainfall rate $p_{I,n}$. In studying the evapotranspiration of a deciduous forest, Hendriks et al. (1990) applied the interception model of Gash (1979), which is based on the approach of Rutter et al. (1971, 1975). In the Rutter approach, weather data on an hourly basis are required, which limits the practical applicability of the model.

Gash (1979) and Mulder (1985) assume three phases in the evaporation of rainwater intercepted by the leaves: (1) *Wetting phase:* the time from the start of the shower to the saturation of the canopy; (2) *Saturation phase:* the time during which the canopy is saturated; (3) *Drying phase:* the time from the end of one shower to the start of the next shower (see Figure 4.5).

Gash (1979) assumes that the evaporation rate and rainfall rate during a shower can be replaced by seasonal averages. The model distinguishes between *small showers,* which do not saturate the canopy, and *large showers,* which do saturate the canopy. The model further assumes that only one shower per day occurs; in other words, that

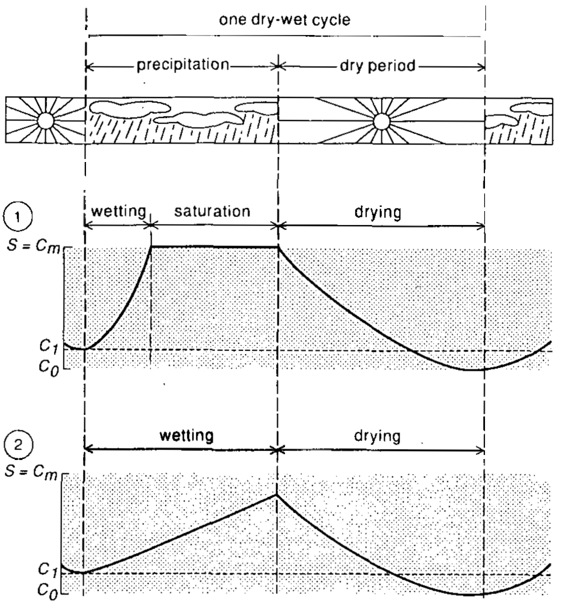

Figure 4.5 Diagram showing change in depth of water on the canopy (C) when saturation is reached (Case 1), or is not reached (Case 2); C_0 is dry canopy, C_1 is partly wet after dry period, C_m is maximum depth of water on the canopy, equal to interception capacity (S) (van Roestel 1984).

the rain depth of a shower equals daily rainfall. The depth of rainfall required to saturate the canopy is calculated as:

$$P_s = -(C_s p_{av}/E_{av}) \ln [1-(E_{av}/p_{av})/(1-f)] \tag{78}$$

where P_s is the depth of rainfall required to saturate the canopy (L), C_s is the canopy storage capacity (L), p_{av} is the seasonal average rainfall intensity (LT^{-1}), E_{av} is the seasonal average evaporation rate of wet canopy during rainfall (LT^{-1}), and f is the free throughfall coefficient (–).

For *large storms*, the interception, E_i, is calculated per phase:

1) Wetting phase: $E_i = P_s(1-f) - C_s$ (79)

2) Saturation phase: $E_i = (E_{av}/p_{av})(P_r-P_s)$ (80)

3) Drying phase: $E_i = C_s$ (81)

For *small storms:*

1) Wetting phase only: $E_i = P_s(1-f)$ (82)