In a step-drawdown test the well is initially pumped at a low constant rate until the drawdown within the well stabilises, i.e. until a steady state is reached. The pumping rate is then increased to a higher constant rate and the well is pumped until the drawdown stabilises once more (Figure 5.1). This process is repeated through at least three steps, which should be of equal duration (say, a few hours each).

5.1 Well and aquifer losses

The drawdown in a pumped well consists of two components: the aquifer losses and the well losses (Figure 5.2). Aquifer losses are the head losses that occur in the aquifer where the flow is laminar. They are time-dependent and vary linearly with the well discharge. The drawdown \( s_1 \) corresponding to this linear aquifer loss can be expressed as

\[
s_1 = B_{1(r_w,t)}Q
\]  

(5.1)
where $B_1$ is the linear aquifer loss coefficient in $\text{d/m}^2$ and $r_w$ is the effective radius of the well. This coefficient can be calculated from the well-flow equations presented in Chapter 4. For confined aquifers for example, it can be expressed using Equations 4.1 and 4.2 as

$$B_1(r_w, t) = \frac{W(u)}{4\pi KH}$$

where $u = (r_w^2 S)/(4KHt)$. From the results of aquifer-test analyses, the values for transmissivity $KH$ and storativity $S$ can be used to calculate $B_1$ values as function of $r_w$ and $t$.

Well losses are divided into linear and non-linear head losses. Linear well losses are caused by the aquifer being damaged during the drilling and completion of the well. They comprise, for example, head losses resulting from the aquifer material compacting during drilling; head losses resulting from the aquifer becoming plugged with drilling mud, which reduces the permeability near the bore hole; head losses in the gravel pack; and head losses in the
The drawdown $s_2$ corresponding to linear well losses can be expressed as

$$s_2 = B_2 Q$$  \hspace{1cm} (5.2)

where $B_2$ is the linear well loss coefficient in d/m². The non-linear well losses include the friction losses that occur inside the well screen and in the suction pipe where the flow is turbulent, and head losses that occur in the zone adjacent to the well where the flow is also usually turbulent. The drawdown $s_3$ corresponding to these non-linear well losses can be expressed as

$$s_3 = C Q^P$$  \hspace{1cm} (5.3)

where $C$ is the non-linear well loss coefficient in dP/m^3⁻¹, and $P$ is an exponent. The general equation describing the drawdown in a pumped well as a function of aquifer/well losses and discharge thus reads

$$s_w = s_1 + s_2 + s_3 = (B_1 + B_2) Q + C Q^P = B Q + C Q^P$$  \hspace{1cm} (5.4)

Jacob (1947) used a constant value of 2 for the exponent $P$. According to Lennox (1966) the value of $P$ can vary between 1.5 and 3.5. Our experience is that in fractured rock aquifers its value may even exceed 3.5. The value of $P = 2$ as proposed by Jacob is, however, still widely accepted. The values of the three parameters $B$, $C$ and $P$ in Equation 5.4 can be found from the analysis of step-drawdown tests. Note that $B$ represents the contribution of the aquifer loss plus the linear well loss; their individual contributions can only be determined from a combination of step-drawdown and aquifer test analyses.

5.2 Well efficiency

The relationship between drawdown and discharge can be expressed as the specific capacity of a well, $Q/s_w$, which describes the productivity of both the aquifer and the well. The specific capacity decreases as pumping continues and also with increasing $Q$. The well efficiency, $E_w$, is defined as the ratio of the aquifer head loss to the total head losses; when expressed as a percentage it reads

$$E_w = \frac{100 B_1 Q}{B Q + C Q^P}$$  \hspace{1cm} (5.5)

The well efficiency according to Equation 5.5 can be assessed if the results of a step-drawdown and of an aquifer test are available. The former are needed for the values of $B$, $C$ and $P$ and the latter for the value of $B_1$. 

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If only the results of a step-drawdown tests are available, the substitution of the B value into Equation 5.5 for the $B_1$ value will overestimate the well efficiency, because $B > B_1$. For these cases, Driscoll (1986) introduced another parameter, $L_p$, being the ratio of the laminar head loss to the total head losses; when expressed as a percentage it reads

$$L_p = \frac{100BQ}{BQ + CQ^p}$$

Equation 5.6 can be used to analyse the well performance yearly, because step-drawdown tests are usually conducted as single-well tests, i.e. the drawdown is not observed in any piezometer. Note that Equation 5.6 is not representative for the well efficiency.

5.3 Diagnostic plots

The analysis of step-drawdown tests uses diagnostic plots in which values of $s_w/Q$ versus $Q$ are plotted on linear paper, with $s_w$ representing the stabilised drawdown at the end of each step. Various configurations of diagnostic plots are possible:

- The points form a horizontal line; this implies that $s_w/Q = B$. Hence, there are no non-linear well losses ($C = 0$). This situation is only encountered at

![Figure 5.3 Time-drawdown plot of field data from a step-drawdown test](image)
very low pumping rates. If the pumping rates are increased, the well will act differently.

- The points form a straight line under a slope; this means that $P = 2$. For this case, Jacob (1947) developed an analysis method to calculate B and C.
- The points form a curved line, i.e. $P$ is unequal to 2. If the curve is concave, $P > 2$; if it is convex, $P < 2$. For these cases, Rorabaugh (1953) developed an analysis method to calculate B, C and $P$.

Under field conditions, the condition that the $s_w$ values used in these plots represent the stabilised drawdowns at the end of each step is not always met (see Figure 5.3). When this occurs, the observed drawdown values at the end of each step need to be corrected before a diagnostic plot can be made. This can be done using the following procedure as developed by Hantush-Bierschenk (Hantush 1964).

**Hantush-Bierschenk’s procedure:**
- Plot the drawdown values of $s_w(t)$ versus the corresponding time $t$ on semi-log paper for all the steps (with $t$ on the logarithmic scale);
- Select a time range in each step where the plots exhibit a straight line and draw the best-fitting straight line through these plotted points;
- Extrapolate the straight line to the end of the next step;
- Determine the increments of drawdown $\Delta s_{w(i)}$ for each step by taking the difference between the observed drawdown at a fixed interval $\Delta t$, taken from the beginning of each step, and the corresponding drawdown on the straight line extrapolated from the preceding step;
- Determine the values of $s_w(n)$ corresponding to the discharge $Q_n$ from $s_w(n) = \Delta s_{w(1)} + \Delta s_{w(2)} + ... + \Delta s_{w(n)}$, where $n = 1, 2, ..., N$ and N the number of steps.

It will be clear that for the drawdown at the end of the first step, $\Delta s_{w(1)}$, no correction can be made with this procedure, i.e. it may represent a non-stabilised drawdown.

### 5.4 Jacob’s method

The values of B and C can be found directly from the diagnostic plot of $s_w/Q$ against $Q$, when $P = 2$. Equation 5.4 then reads

$$s_w/Q = B + C Q$$

Equation 5.7 implies that a plot of $s_w/Q$ versus $Q$ on linear paper would yield a straight line under a slope (Figure 5.4). The slope of this straight line is equal to C, while the value of B can be found by extending the straight line until it intercepts the $Q = 0$ axis.
The Jacob method is applicable in any type of aquifer if the following assumptions and conditions are satisfied:
- The assumptions listed at the beginning of Chapter 4, with the exception of the fourth assumption, which is replaced by: the aquifer is pumped step-wise at increased discharge rates.

The following conditions are added:
- The pumped well penetrates the entire thickness of the aquifer and thus receives water by horizontal flow;
- The non-linear well losses are appreciable and vary according to the expression \( CQ^2 \).

**Procedure**
- Plot the drawdown values of \( s_w(t) \) versus the corresponding time \( t \) on semi-log paper (\( t \) on logarithmic scale);
- If the drawdowns at the end of each step did not stabilise, apply the Hantush-Bierschenk procedure and correct the values of \( s_{w(n)} \);
- On linear paper, plot the values of \( s_{w(n)}/Q_n \) versus the corresponding values of \( Q_n \);
- Fit a straight line through the plotted points;
- Determine the slope of the straight line \( \Delta(s_w/Q)/\Delta Q \), which represents the value of \( C \);
- Extend the straight line until it intercepts the \( Q = 0 \) axis. The \( s_w/Q \) value of the interception point represents the value of \( B \).
5.5 Rorabaugh’s method

The values of $B$, $C$, and $P$ cannot be found directly from the diagnostic plot of $s_w/Q$ versus $Q$ itself, when $P$ is unequal to 2. Equation 5.4 then reads

$$s_w/Q = B + CQ^{P-1}$$  \hfill (5.8)

Figure 5.5 Log-log plot of $[s_w/Q - B]$ versus $Q$ of field data from a step-drawdown test analysed with the Rorabaugh method.
Rearranging Equation 5.8 and taking the logarithms, it can also be written as

\[ \log[s_w/Q - B] = \log C + (P-1)\log Q \]  

(5.9)

Equation 5.9 implies that a plot of \([s_w/Q - B]\) versus \(Q\) on log-log paper would yield a straight line under a slope. The slope of this straight line is equal to \(P-1\), while the value of \(C\) can be found by extending the straight line until it intercepts the \(Q = 1\) axis.

The Rorabaugh method is applicable in any type of aquifer if the following assumptions and conditions are satisfied:

- The assumptions listed at the beginning of Chapter 4, with the exception of the fourth assumption, which is replaced by: the aquifer is pumped step-wise at increased discharge rates.

The following conditions are added:

- The pumped well penetrates the entire thickness of the aquifer and thus receives water by horizontal flow;
- The non-linear well losses are appreciable and vary according to the expression \(CQ^P\).

**Procedure**

- Plot the drawdown values of \(s_w(t)\) versus the corresponding time \(t\) on semi-log paper (with \(t\) on the logarithmic scale);
- If the drawdowns at the end of each step did not stabilise, apply the Hantush-Bierschenk procedure and correct the values of \(s_w(n)\);
- On linear paper, plot the values of \(s_w/n\) versus the corresponding values of \(Q_n\);
- Fit a curved line through the plotted points;
- Extend the curved line smoothly until it intercepts the \(Q = 0\) axis. Take the interception point on this \(s_w/Q\) axis as initial estimate \(B_i\) of \(B\);
- Calculate \([s_w(n)/Q_n - B_i]\) for each step;
- Plot the values of \([s_w(n)/Q_n - B_i]\) versus \(Q_n\) on log-log paper. Repeat this part of the procedure for different values of \(B_i\). The value of \(B_i\) that gives the straight line on the plot will be the correct value of \(B\). Figure 5.5 shows that the data points are located on a straight line for a \(B\) value of \(1.0 \times 10^{-3}\);
- Calculate the slope of the straight line \(\log \Delta[(s_w/Q) - B]/\log \Delta Q\). This equals \((P-1)\), from which \(P\) can be obtained;
- Extend the straight line until it intercepts the \(Q = 1\) axis. This value of \([s_w/Q) - B]\) represents the value of \(C\).